Assignment 1

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Classical Mechanics II

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Problem 1

[20 pts] Spiral trajectory

A point particle is moving in the xy-plane parameterized by φ as follows:

$$x(\varphi) = a\varphi\cos\varphi, \quad y(\varphi) = a\varphi\sin\varphi$$

, where $a > 0$ and $\varphi \geq 0$. Assume that the particle moves along the trajectory above with $\varphi(t) = \alpha t$ where α is a constant.

(a) Calculate the Cartesian components of the velocity $\boldsymbol{v}$, acceleration $\boldsymbol{a}$, the radius of curvature ρ and the radius of torsion $\sigma \equiv \frac{1}{\tau}$ as a function of time t.

(b) Calculate the speed v as functions of time t.

Solution

Part (a): Velocity and acceleration

The position vector in polar form is:

$$\boldsymbol{r}(\varphi) = a\varphi\cos\varphi\hat{e}_x + a\varphi\sin\varphi\hat{e}_y = a\varphi\hat{e}_r$$

Using the chain rule with $\varphi = \alpha t$:

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\varphi}\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\varphi}\alpha$$

In polar coordinates, $\boldsymbol{r} = r\hat{e}_r$ where $r = a\varphi$:

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\varphi} = \frac{\mathrm{d}r}{\mathrm{d}\varphi}\hat{e}_r + r\frac{\mathrm{d}\hat{e}_r}{\mathrm{d}\varphi} = a\hat{e}_r + a\varphi\hat{e}_\varphi$$

Therefore:

$$\boldsymbol{v} = \alpha\left(a\hat{e}_r + a\varphi\hat{e}_\varphi\right) = a\alpha\left(\hat{e}_r + \varphi\hat{e}_\varphi\right)$$

For acceleration:

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}\varphi}\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \alpha\frac{\mathrm{d}}{\mathrm{d}\varphi}\left[a\alpha\left(\hat{e}_r + \varphi\hat{e}_\varphi\right)\right]$$

$$= a\alpha^2\left[\frac{\mathrm{d}\hat{e}_r}{\mathrm{d}\varphi} + \hat{e}_\varphi + \varphi\frac{\mathrm{d}\hat{e}_\varphi}{\mathrm{d}\varphi}\right]$$

Using $\frac{\mathrm{d}\hat{e}_r}{\mathrm{d}\varphi} = \hat{e}_\varphi$ and $\frac{\mathrm{d}\hat{e}_\varphi}{\mathrm{d}\varphi} = -\hat{e}_r$:

$$\boldsymbol{a} = a\alpha^2\left[\hat{e}_\varphi + \hat{e}_\varphi - \varphi\hat{e}_r\right] = a\alpha^2\left[\left(2\hat{e}_\varphi - \varphi\hat{e}_r\right)\right]$$

Converting to Cartesian coordinates with $\varphi = \alpha t$, $\hat{e}_r = \cos\varphi\hat{e}_x + \sin\varphi\hat{e}_y$, and $\hat{e}_\varphi = -\sin\varphi\hat{e}_x + \cos\varphi\hat{e}_y$:

$$\boxed{\boldsymbol{v} = a\alpha\left[(\cos\alpha t - \alpha t\sin\alpha t)\hat{e}_x + (\sin\alpha t + \alpha t\cos\alpha t)\hat{e}_y\right]}$$

$$\boxed{\boldsymbol{a} = a\alpha^2\left[(-\alpha t\cos\alpha t - 2\sin\alpha t)\hat{e}_x + (-\alpha t\sin\alpha t + 2\cos\alpha t)\hat{e}_y\right]}$$

Radius of curvature

The curvature κ is given by:

$$\kappa = \frac{|\boldsymbol{v}\times\boldsymbol{a}|}{|\boldsymbol{v}|^3}$$

Computing the cross product (in 2D, we extend to 3D):

$$\boldsymbol{v}\times\boldsymbol{a} = \begin{vmatrix} \hat{e}_x & \hat{e}_y & \hat{e}_z \\ v_x & v_y & 0 \\ a_x & a_y & 0 \end{vmatrix} = \left(v_x a_y - v_y a_x\right)\hat{e}_z$$

In polar form this is simpler:

$$\boldsymbol{v}\times\boldsymbol{a} = a\alpha\left(\hat{e}_r + \varphi\hat{e}_\varphi\right)\times a\alpha^2\left(2\hat{e}_\varphi - \varphi\hat{e}_r\right)$$

Using $\hat{e}_r\times\hat{e}_\varphi = \hat{e}_z$ and $\hat{e}_\varphi\times\hat{e}_r = -\hat{e}_z$:

$$\boldsymbol{v}\times\boldsymbol{a} = a^2\alpha^3\left[\left(\hat{e}_r\times 2\hat{e}_\varphi\right) + \left(\varphi\hat{e}_\varphi\times(-\varphi\hat{e}_r)\right)\right] = a^2\alpha^3(2+\varphi^2)\hat{e}_z$$

The magnitude is:

$$|\boldsymbol{v}\times\boldsymbol{a}| = a^2\alpha^3(2+\varphi^2) = a^2\alpha^3(2+\alpha^2t^2)$$

The speed is:

$$|\boldsymbol{v}|^2 = a^2\alpha^2(1+\varphi^2) = a^2\alpha^2(1+\alpha^2t^2)$$

Therefore:

$$\kappa = \frac{a^2\alpha^3(2+\alpha^2t^2)}{[a^2\alpha^2(1+\alpha^2t^2)]^{\frac{3}{2}}} = \frac{\alpha(2+\alpha^2t^2)}{a(1+\alpha^2t^2)^{\frac{3}{2}}}$$

The radius of curvature is:

$$\boxed{\rho = \frac{1}{\kappa} = \frac{a(1+\alpha^2t^2)^{\frac{3}{2}}}{\alpha(2+\alpha^2t^2)}}$$

Radius of torsion

For a planar curve (motion confined to xy-plane), the torsion $\tau = 0$ everywhere since the curve never leaves the plane. Therefore:

$$\boxed{\sigma = \frac{1}{\tau} = \infty}$$

The infinite radius of torsion indicates no twisting out of the plane.

Part (b): Speed

From our calculation above:

$$v = |\boldsymbol{v}| = a\alpha\sqrt{1+\varphi^2} = a\alpha\sqrt{1+\alpha^2 t^2}$$

$$\boxed{v(t) = a\alpha\sqrt{1+\alpha^2 t^2}}$$

The initial speed is $v(0) = a\alpha$, and the speed increases monotonically with time as the particle spirals outward.

Problem 2

[20 pts] Projectile with quadratic drag

A particle is projected vertically upwards with speed u_0 and moves under uniform gravity in a medium that exerts a resistance force proportional to the square of its speed and in which the particle's terminal speed is V_∞.

(a) Find the maximum height above the starting point attained by the particle and the time taken to reach that height.

(b) Show also that the speed of the particle when it returns to its starting point is $\sqrt{\frac{u_0 V_\infty}{u_0^2+V_\infty^2}}$.

Solution

Setup

For vertical motion with upward positive, the equation of motion is:

$$m\frac{\mathrm{d}v}{\mathrm{d}t} = -mg - kv^2$$

The terminal velocity occurs when $mg = kV_\infty^2$, giving $k = \frac{mg}{V_\infty^2}$. Substituting:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -g\left(1+\frac{v^2}{V_\infty^2}\right) = -\left(\frac{g}{V_\infty^2}\right)(V_\infty^2 + v^2)$$

Part (a): Maximum height and time

Using the chain rule $\frac{\mathrm{d}v}{\mathrm{d}t} = v\frac{\mathrm{d}v}{\mathrm{d}y}$:

$$v\frac{\mathrm{d}v}{\mathrm{d}y} = -\left(\frac{g}{V_\infty^2}\right)(V_\infty^2 + v^2)$$

Separating variables:

$$v\,\mathrm{d}\frac{v}{V_\infty^2 + v^2} = -\left(\frac{g}{V_\infty^2}\right)\mathrm{d}y$$

Integrating from initial conditions $(y_0 = 0, v_0 = u_0)$ to general (y, v):

$$\frac{1}{2}\ln\left(\frac{V_\infty^2 + v^2}{V_\infty^2 + u_0^2}\right) = -\frac{gy}{V_\infty^2}$$

$$V_\infty^2 + v^2 = (V_\infty^2 + u_0^2)e^{-2g\frac{y}{V_\infty^2}}$$

At maximum height, $v = 0$:

$$V_\infty^2 = (V_\infty^2 + u_0^2)e^{-2g\frac{h_{\max}}{V_\infty^2}}$$

Solving for $h_{\max}$:

$$e^{2g\frac{h_{\max}}{V_\infty^2}} = \frac{V_\infty^2 + u_0^2}{V_\infty^2} = 1 + \frac{u_0^2}{V_\infty^2}$$

$$\boxed{h_{\max} = \frac{V_\infty^2}{2g} \ln\left(1 + \frac{u_0^2}{V_\infty^2}\right)}$$

For the time, we use:

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\left(\frac{g}{V_\infty^2}\right)(V_\infty^2 + v^2)$$

Separating variables:

$$\frac{\mathrm{d}v}{V_\infty^2 + v^2} = -\left(\frac{g}{V_\infty^2}\right)\mathrm{d}t$$

$$\frac{1}{V_\infty} \arctan\left(\frac{v}{V_\infty}\right) \Big|_{u_0}^{0} = -\left(\frac{g}{V_\infty^2}\right) t_{\max}$$

$$-\frac{1}{V_\infty} \arctan\left(\frac{u_0}{V_\infty}\right) = -\left(\frac{g}{V_\infty^2}\right) t_{\max}$$

$$\boxed{t_{\max} = \frac{V_\infty}{g} \arctan\left(\frac{u_0}{V_\infty}\right)}$$

Part (b): Return speed

On the way down, taking downward as positive and denoting the downward speed as w:

$$\frac{\mathrm{d}w}{\mathrm{d}t} = g - \left(\frac{g}{V_\infty^2}\right) w^2 = \left(\frac{g}{V_\infty^2}\right)(V_\infty^2 - w^2)$$

Using $\frac{\mathrm{d}w}{\mathrm{d}t} = w\frac{\mathrm{d}w}{\mathrm{d}y}$ where y is measured downward from maximum height:

$$w\frac{\mathrm{d}w}{\mathrm{d}y} = \left(\frac{g}{V_\infty^2}\right)(V_\infty^2 - w^2)$$

$$\frac{w\,\mathrm{d}w}{V_\infty^2 - w^2} = \left(\frac{g}{V_\infty^2}\right)\mathrm{d}y$$

Integrating from $(y = 0, w = 0)$ to $(y = h_{\max}, w = v_{\text{return}})$:

$$-\frac{1}{2} \ln\left(\frac{V_\infty^2 - w^2}{V_\infty^2}\right) = \frac{gy}{V_\infty^2}$$

At $y = h_{\max}$:

$$-\frac{1}{2} \ln\left(\frac{V_\infty^2 - v_{\text{return}}^2}{V_\infty^2}\right) = \frac{gh_{\max}}{V_\infty^2}$$

$$\ln\left(\frac{V_\infty^2 - v_{\text{return}}^2}{V_\infty^2}\right) = -2g\frac{h_{\max}}{V_\infty^2} = -\ln\left(1 + \frac{u_0^2}{V_\infty^2}\right)$$

$$\frac{V_\infty^2 - v_{\text{return}}^2}{V_\infty^2} = \frac{1}{1 + \frac{u_0^2}{V_\infty^2}} = \frac{V_\infty^2}{V_\infty^2 + u_0^2}$$

$$V_\infty^2 - v_{\text{return}}^2 = \frac{V_\infty^4}{V_\infty^2 + u_0^2}$$

$$v_{\text{return}}^2 = V_\infty^2 - \frac{V_\infty^4}{V_\infty^2 + u_0^2} = \frac{V_\infty^2(V_\infty^2 + u_0^2) - V_\infty^4}{V_\infty^2 + u_0^2} = \frac{V_\infty^2 u_0^2}{V_\infty^2 + u_0^2}$$

$$\boxed{v_{\text{return}} = \frac{V_\infty u_0}{\sqrt{V_\infty^2 + u_0^2}}}$$

The return speed is always less than the launch speed u_0 due to energy dissipation by air resistance. In the limit $V_\infty \to \infty$ (no drag), $v_{\text{return}} \to u_0$, recovering energy conservation.

Problem 3

[30 pts] Electron in crossed electric and magnetic fields

An electron of mass m and charge $-e$ is moving under the combined influence of a uniform electric field $E_0\hat{e}_y$ and a uniform magnetic field $B_0\hat{e}_z$. Initially, the electron is at the origin and is moving with velocity $u_0\hat{e}_x$. Find the trajectory, $x(t)$, $y(t)$, $z(t)$, of the electron in its subsequent motion.

Remark: The general path is called a trochoid which becomes a cycloid in the special case. Cycloidal motion of motion of electrons is used in the magnetron vacuum tube which generates the microwaves in a microwave oven.

Solution

Equation of motion

The Lorentz force on the electron is:

$$\boldsymbol{F} = -e(\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B})$$

$$m\boldsymbol{a} = -e\left(E_0\hat{e}_y + \boldsymbol{v} \times B_0\hat{e}_z\right)$$

Expanding the cross product:

$$\boldsymbol{v} \times \boldsymbol{B} = \left(\dot{x}\hat{e}_x + \dot{y}\hat{e}_y + \dot{z}\hat{e}_z\right) \times B_0\hat{e}_z = B_0\left(\dot{x}\hat{e}_y - \dot{y}\hat{e}_x\right)$$

Therefore:

$$m\boldsymbol{a} = -e\left[E_0\hat{e}_y + B_0\dot{x}\hat{e}_y - B_0\dot{y}\hat{e}_x\right]$$

$$m\boldsymbol{a} = -e\left[(-B_0\dot{y})\hat{e}_x + (E_0 + B_0\dot{x})\hat{e}_y\right]$$

Component equations:

$$\begin{cases} m\ddot{x} = eB_0\dot{y} \\ m\ddot{y} = -e(E_0 + B_0\dot{x}) \\ m\ddot{z} = 0 \end{cases}$$

Define the cyclotron frequency $\omega_c \equiv \frac{eB_0}{m} > 0$:

$$\begin{cases} \ddot{x} = \omega_c \dot{y} \\ \ddot{y} = -\dfrac{eE_0}{m} - \omega_c \dot{x} \\ \ddot{z} = 0 \end{cases}$$

Solving the z-component

From $\ddot{z} = 0$ with initial conditions $z(0) = 0$, $\dot{z}(0) = 0$:

$$\boxed{z(t) = 0}$$

The motion is confined to the xy-plane.

Solving the coupled xy system

Differentiating the first equation:

$$\dddot{x} = \omega_c \ddot{y} = \omega_c \left[-\frac{eE_0}{m} - \omega_c \dot{x} \right] = -\frac{e\omega_c E_0}{m} - \omega_c^2 \dot{x}$$

This gives:

$$\dddot{x} + \omega_c^2 \dot{x} = -\frac{e\omega_c E_0}{m}$$

The homogeneous solution is:

$$\dot{x}_h = A \cos \omega_c t + B \sin \omega_c t$$

For the particular solution, try a constant: $\dot{x}_p = C$:

$$0 + \omega_c^2 C = -\frac{e\omega_c E_0}{m} \Longrightarrow C = -\frac{eE_0}{m\omega_c} = -\frac{E_0}{B_0}$$

General solution for velocity:

$$\dot{x} = A \cos \omega_c t + B \sin \omega_c t - \frac{E_0}{B_0}$$

From $\ddot{x} = \omega_c \dot{y}$:

$$-A\omega_c \sin \omega_c t + B\omega_c \cos \omega_c t = \omega_c \dot{y}$$
$$\dot{y} = -A \sin \omega_c t + B \cos \omega_c t$$

Applying initial conditions

At $t = 0$: $\dot{x}(0) = u_0$, $\dot{y}(0) = 0$:

$$\begin{cases} A - \frac{E_0}{B_0} = u_0 \\ B = 0 \end{cases} \Longrightarrow \begin{cases} A = u_0 + \dfrac{E_0}{B_0} \\ B = 0 \end{cases}$$

Therefore:

$$\begin{cases} \dot{x} = \left(u_0 + \dfrac{E_0}{B_0} \right) \cos \omega_c t - \dfrac{E_0}{B_0} \\ \dot{y} = -\left(u_0 + \dfrac{E_0}{B_0} \right) \sin \omega_c t \end{cases}$$

Integrating for position

With $x(0) = y(0) = 0$:

$$x(t) = \int_0^t \left[\left(u_0 + \frac{E_0}{B_0} \right) \cos \omega_c t' - \frac{E_0}{B_0} \right] \mathrm{d}t'$$

$$= \left(u_0 + \frac{E_0}{B_0} \right) \frac{\sin \omega_c t}{\omega_c} - \frac{E_0 t}{B_0}$$

$$y(t) = \int_0^t \left[- \left(u_0 + \frac{E_0}{B_0} \right) \sin \omega_c t' \right] \mathrm{d}t'$$

$$= \left(u_0 + \frac{E_0}{B_0} \right) \frac{1 - \cos \omega_c t}{\omega_c}$$

Final trajectory

$$\boxed{\begin{cases} x(t) = \dfrac{u_0 B_0 + E_0}{\omega_c B_0} \sin \omega_c t - \dfrac{E_0 t}{B_0} \\[2ex] y(t) = \dfrac{u_0 B_0 + E_0}{\omega_c B_0} (1 - \cos \omega_c t) \\[2ex] z(t) = 0 \end{cases}}$$

where $\omega_c = \frac{eB_0}{m}$.

The trajectory consists of cyclotron motion (circular motion due to the magnetic field) with center drifting in the $-x$ direction. The drift velocity is $v_{\text{drift}} = \frac{E_0}{B_0}$, which is the $\boldsymbol{E} \times \boldsymbol{B}$ drift. The radius of the cyclotron orbit is:

$$R = \frac{u_0 B_0 + E_0}{\omega_c B_0} = \frac{m(u_0 B_0 + E_0)}{eB_0^2}$$

Problem 4

[30 pts] Block sliding on helical track

A small block of mass m glides under its own weight $\boldsymbol{W} = -mg\hat{e}_z$ frictionless downward along a helical track

$$\boldsymbol{r}(t) = a\cos\varphi(t)\hat{e}_x + a\sin\varphi(t)\hat{e}_y + b\varphi(t)\hat{e}_z$$

, where a and b are positive constants. The block starts its motion with $\varphi(0) = \varphi_0$ and $\dot{\varphi}(0) = 0$.

(a) Derive a second order ordinary differential equation for $\varphi(t)$ governing the dynamics of the block. Solve for $\varphi(t)$ and calculate the magnitude of the velocity $\boldsymbol{v}(t)$ of the block as a function of time.

(b) Calculate the magnitude of the force $\boldsymbol{F}(t)$ exerted on the block by the helical track as a function of time.

Solution

Part (a): Equation of motion

The position vector is:

$$\boldsymbol{r} = a\cos\varphi\hat{e}_x + a\sin\varphi\hat{e}_y + b\varphi\hat{e}_z$$

The velocity is:

$$\boldsymbol{v} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\varphi}\dot{\varphi} = \left[-a\sin\varphi\hat{e}_x + a\cos\varphi\hat{e}_y + b\hat{e}_z\right]\dot{\varphi}$$

The speed is:

$$v^2 = a^2\dot{\varphi}^2 + b^2\dot{\varphi}^2 = (a^2+b^2)\dot{\varphi}^2$$

The acceleration is:

$$\boldsymbol{a} = \frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = \left[-a\sin\varphi\hat{e}_x + a\cos\varphi\hat{e}_y + b\hat{e}_z\right]\ddot{\varphi} + \left[-a\cos\varphi\hat{e}_x - a\sin\varphi\hat{e}_y\right]\dot{\varphi}^2$$

Energy method

Since the track is frictionless, we use conservation of energy. Taking the initial height as reference ($z = b\varphi_0$):

$$E = \frac{1}{2}mv^2 + mgz = \frac{1}{2}m(a^2+b^2)\dot{\varphi}^2 + mgb\varphi$$

At $t = 0$: $E = mgb\varphi_0$ (since $\dot{\varphi}(0) = 0$)

Therefore:

$$\frac{1}{2}m(a^2+b^2)\dot{\varphi}^2 + mgb\varphi = mgb\varphi_0$$

$$\dot{\varphi}^2 = \frac{2gb}{a^2+b^2}(\varphi_0 - \varphi)$$

Taking the time derivative:

$$2\dot{\varphi}\ddot{\varphi} = -\frac{2gb}{a^2+b^2}\dot{\varphi}$$

For $\dot{\varphi} \neq 0$:

$$\boxed{\ddot{\varphi} = -\frac{gb}{a^2+b^2}}$$

This is constant acceleration! Solving with initial conditions $\varphi(0) = \varphi_0$, $\dot{\varphi}(0) = 0$:

$$\dot{\varphi}(t) = -\frac{gb}{a^2+b^2}t$$

$$\varphi(t) = \varphi_0 - \frac{1}{2}\frac{gb}{a^2+b^2}t^2$$

The mass descends, so φ decreases with time (note the negative sign is consistent since $\dot{\varphi} < 0$).

The speed is:

$$|\boldsymbol{v}(t)| = \sqrt{a^2+b^2}\,|\dot{\varphi}| = \sqrt{a^2+b^2}\frac{gb}{a^2+b^2}t$$

$$\boxed{\begin{cases} \varphi(t) = \varphi_0 - \dfrac{gb}{2(a^2+b^2)}t^2 \\ |\boldsymbol{v}(t)| = \dfrac{gb}{\sqrt{a^2+b^2}}t \end{cases}}$$

Part (b): Normal force

The forces on the mass are gravity $\boldsymbol{F}_g = -mg\hat{e}_z$ and the normal force $\boldsymbol{N}$ perpendicular to the track.

Newton's second law:

$$m\boldsymbol{a} = \boldsymbol{F}_g + \boldsymbol{N}$$

The normal force is perpendicular to the velocity. The tangent to the helix is:

$$\hat{t} = \frac{\boldsymbol{v}}{|\boldsymbol{v}|} = \frac{[-a\sin\varphi\hat{e}_x + a\cos\varphi\hat{e}_y + b\hat{e}_z]}{\sqrt{a^2+b^2}}$$

The component of gravity along the track:

$$(\boldsymbol{F}_g\cdot\hat{t})\hat{t} = \left((-mg\hat{e}_z)\cdot\frac{[-a\sin\varphi\hat{e}_x + a\cos\varphi\hat{e}_y + b\hat{e}_z]}{\sqrt{a^2+b^2}}\right)\hat{t}$$

$$= -\frac{mgb}{\sqrt{a^2+b^2}}\hat{t}$$

The tangential component of Newton's law gives:

$$ma_{\tan} = m\frac{\mathrm{d}v}{\mathrm{d}t} = -\frac{mgb}{\sqrt{a^2+b^2}}$$

This is consistent with our energy equation: $\frac{\mathrm{d}v}{\mathrm{d}t} = \sqrt{a^2+b^2}\ddot{\varphi} = -\frac{gb}{\sqrt{a^2+b^2}}$.

The normal force satisfies:

$$\boldsymbol{N} = m\boldsymbol{a} - \boldsymbol{F}_g$$

Computing the acceleration with $\ddot{\varphi} = -\frac{gb}{a^2+b^2}$ and $\dot{\varphi} = -\frac{gbt}{a^2+b^2}$:

$$\boldsymbol{a} = [-a\sin\varphi\hat{e}_x + a\cos\varphi\hat{e}_y + b\hat{e}_z]\left(-\frac{gb}{a^2+b^2}\right) + [-a\cos\varphi\hat{e}_x - a\sin\varphi\hat{e}_y]\left(\frac{gbt}{a^2+b^2}\right)^2$$

$$\boldsymbol{N} = m\boldsymbol{a} + mg\hat{e}_z$$

$$= m\left[-\frac{gb}{a^2+b^2}\left(-a\sin\varphi\hat{e}_x + a\cos\varphi\hat{e}_y + b\hat{e}_z + \left(\frac{gbt}{a^2+b^2}\right)^2\left(-a\cos\varphi\hat{e}_x - a\sin\varphi\hat{e}_y\right)\right] + mg\hat{e}_z\right]$$

The z-component:

$$N_z = m\left[-\frac{gb}{a^2+b^2}b + g\right] = mg\left[1 - \frac{b^2}{a^2+b^2}\right] = \frac{mga^2}{a^2+b^2}$$

The horizontal components:

$$N_x = m\left[\frac{agb\sin\varphi}{a^2+b^2} - \frac{ag^2b^2t^2\cos\varphi}{(a^2+b^2)^2}\right]$$

$$N_y = m\left[-\frac{agb\cos\varphi}{a^2+b^2} - \frac{ag^2b^2t^2\sin\varphi}{(a^2+b^2)^2}\right]$$

Computing $N_x^2 + N_y^2$:

$$N_x^2 = m^2\left[\frac{(agb\sin\varphi)^2}{(a^2+b^2)^2} - \frac{2a^2g^3b^3t^2\sin\varphi\cos\varphi}{(a^2+b^2)^3} + \frac{a^2g^4b^4t^4\cos^2\varphi}{(a^2+b^2)^4}\right]$$

$$N_y^2 = m^2\left[\frac{(agb\cos\varphi)^2}{(a^2+b^2)^2} + \frac{2a^2g^3b^3t^2\sin\varphi\cos\varphi}{(a^2+b^2)^3} + \frac{a^2g^4b^4t^4\sin^2\varphi}{(a^2+b^2)^4}\right]$$

Adding these (note the cross terms cancel):

$$N_x^2 + N_y^2 = m^2\left[\frac{(agb)^2}{(a^2+b^2)^2}(\sin^2\varphi + \cos^2\varphi) + \frac{a^2g^4b^4t^4}{(a^2+b^2)^4}(\cos^2\varphi + \sin^2\varphi)\right]$$

$$= m^2\left[\frac{(agb)^2}{(a^2+b^2)^2} + \frac{a^2g^4b^4t^4}{(a^2+b^2)^4}\right]$$

$$= \frac{m^2a^2g^2b^2}{(a^2+b^2)^2}\left[1 + \frac{g^2b^2t^4}{(a^2+b^2)^2}\right]$$

Therefore:

$$|\boldsymbol{N}|^2 = N_x^2 + N_y^2 + N_z^2 = \frac{m^2a^2g^2b^2}{(a^2+b^2)^2}\left[1 + \frac{g^2b^2t^4}{(a^2+b^2)^2}\right] + \frac{m^2g^2a^4}{(a^2+b^2)^2}$$

$$= \frac{m^2g^2a^2}{(a^2+b^2)^2}\left[b^2 + \frac{g^2b^4t^4}{(a^2+b^2)^2} + a^2\right]$$

$$= \frac{m^2g^2a^2}{(a^2+b^2)^2}\left[(a^2+b^2) + \frac{g^2b^4t^4}{(a^2+b^2)^2}\right]$$

$$\boxed{|\boldsymbol{N}(t)| = \frac{mga}{\sqrt{a^2+b^2}}\sqrt{1 + \frac{g^2b^4t^4}{(a^2+b^2)^3}}}$$

Alternatively, factoring differently:

$$\boxed{|\boldsymbol{N}(t)| = mg\sqrt{\frac{a^2}{a^2+b^2} + \frac{a^2 g^2 b^4 t^4}{\left(a^2+b^2\right)^4}}}$$

At $t = 0$:

$$|\boldsymbol{N}(0)| = \frac{mga}{\sqrt{a^2+b^2}}$$

The normal force has a constant vertical component $\frac{mga^2}{a^2+b^2}$ and a horizontal component that grows with time as the mass accelerates down the helix. The centripetal acceleration requires increasing horizontal normal force to keep the mass on the curved track.