Propagation of Laser Light: Gaussian Beam optics

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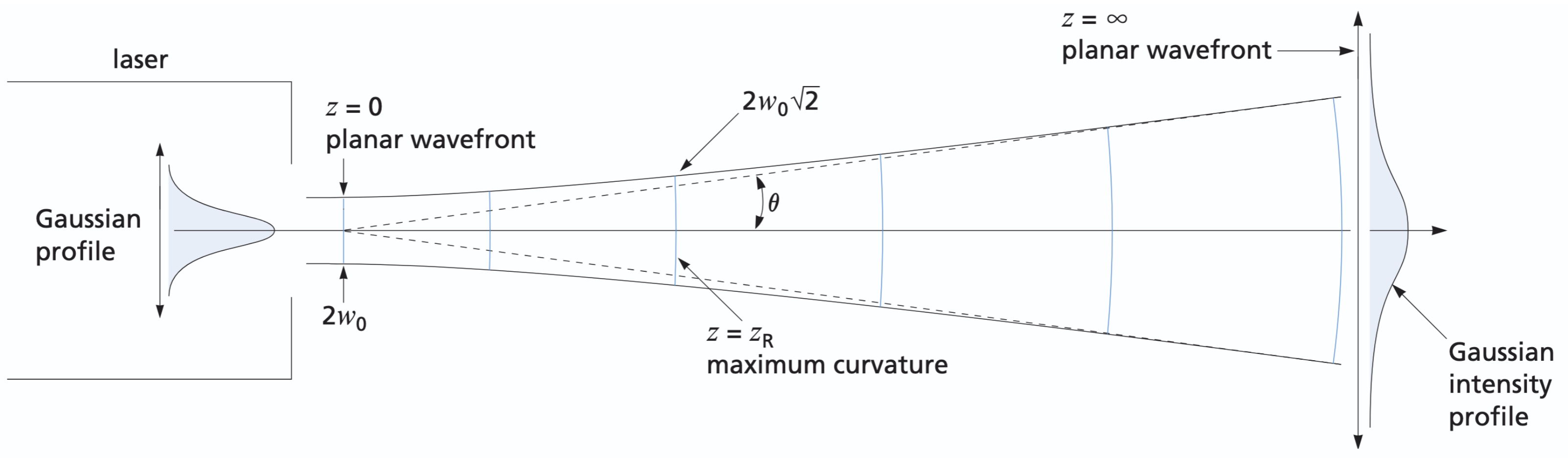


In many cases, a laser beam appears to diverge in a conical shape, but most of the time intensity distribution can be approximated by an ideal Gaussian intensity profile.

The goal of this experiment was to verify whether the given laser follows the Gaussian intensity distribution and to calculate the M^2 value, a key indicator of beam quality.

Introduction

For a laser which emits a beam with a Gaussian profile, the intensity is given by the following equation

$$I_s = I_0 \exp\left(-\frac{2r^2}{W(z)^2}\right), \quad (1)$$

where r is the horizontal displacement from the beam waist and

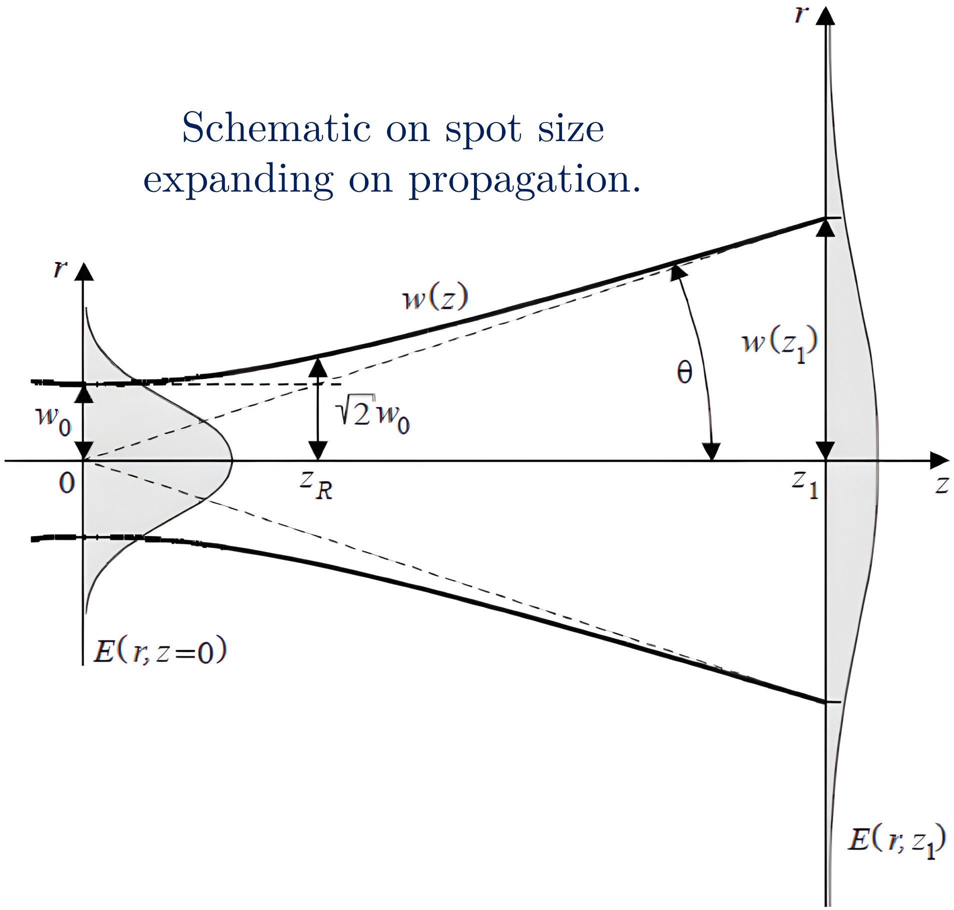
$$W(z) = W_0\sqrt{1+\left(\frac{z}{Z_R}\right)^2}, \quad (2)$$

with W_0 being the beam waist. The parameter

$$Z_R = \frac{\pi W_0^2}{\lambda} \quad (3)$$

is called the Rayleigh range.

Schematic on spot size expanding on propagation.



Legends

$w(z)$: *Spot size / Beam Width*	z_R : *Rayleigh range*
w_0 : *Beam waist*	2θ : *Divergence of beam*

M^2 parameter of laser

For $Z \gg Z_R$, the beam radius can be approximated as

$$W(z) \approx W_0 \frac{Z}{Z_R} = \frac{\lambda}{\pi W_0} Z, \tag{4}$$

so the divergence angle is

$$\theta \approx \frac{2\lambda}{\pi W_0}. \tag{5}$$

The M^2 value is defined through

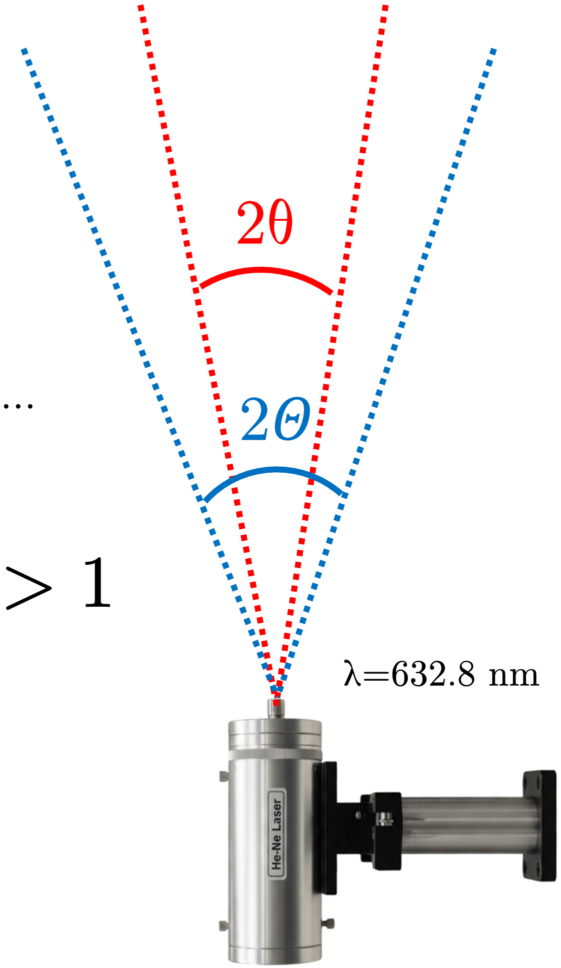
$$\theta = M^2 \frac{2\lambda}{\pi W_0}, \tag{6}$$

which gives

$$M^2 = \frac{\theta Z_R}{W_0} = \frac{\theta \pi W_0}{\lambda}. \tag{7}$$

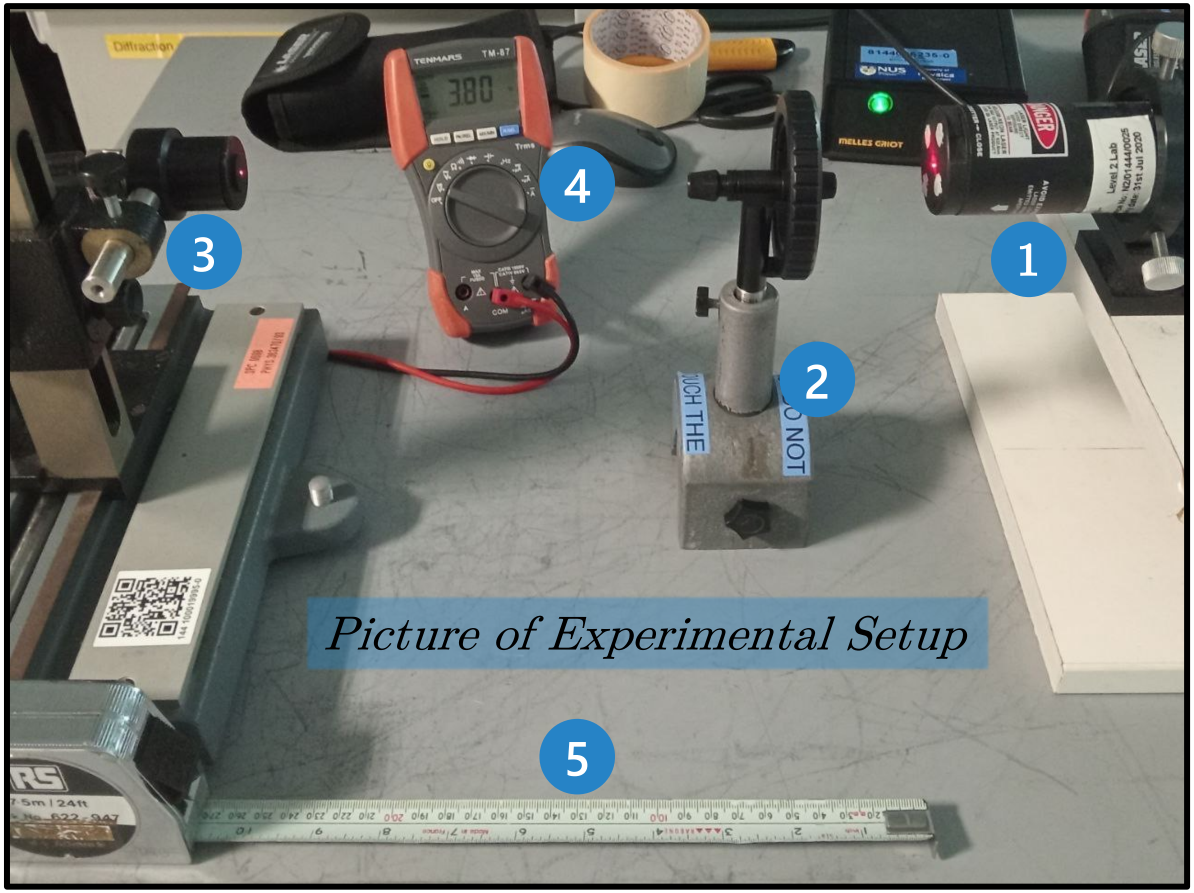
In reality, lasers are not perfect Gaussian beams....

$$\mathrm{M}^2 = \frac{\Theta}{\theta} > 1$$



Legends	2Θ : *Actual Angular Spread*	2θ : *Predicted Divergence*

Experimental Setup



Picture of Experimental Setup

Legends

1 : Laser Source

2 : Attenuator

3 : Detector

4 : Voltmeter

5 : Measuring Tape

<u>Determination of Rayleigh Range</u>

z : 20 cm - 200 cm

<u>Determination of Angular Spread</u>

z : 5 m - 15 m

Raw Data

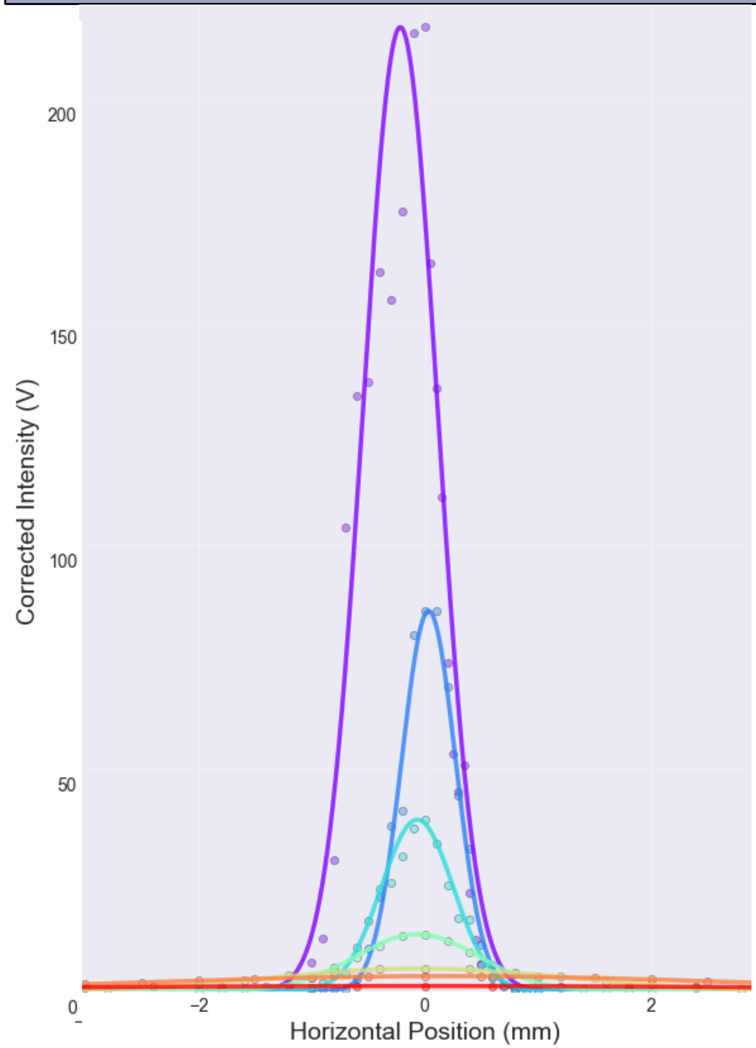
PC2193 Propagation of Laser Light: Gaussian Beam Optics

Experiment 2

	O1	O2	O3	Oavg
Offset value	6.9	6.75	6.8	6.82

		Distance D/cm	Corrected Distance/cm																						
		D	D-Oavg																						
	Attenuator	20.00		Horizontal offset W/mm	-0.50	-0.45	-0.40	-0.35	-0.30	-0.25	-0.20	-0.15	-0.10	-0.05	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
	85º	20.05		Reading 1/V	0.048	0.089	0.226	0.790	1.012	1.035	1.223	1.174	1.325	1.630	1.639	1.237	1.025	0.839	0.559	0.403	0.340	0.385	0.166	0.087	0.049
		20.05		Reading 2/V	0.048	0.090	0.221	0.788	1.011	1.036	1.224	1.176	1.326	1.628	1.638	1.237	1.026	0.841	0.558	0.404	0.340	0.385	0.166	0.087	0.044
		20.00		Reading 3/V	0.048	0.090	0.221	0.789	1.012	1.035	1.222	1.177	1.328	1.630	1.640	1.238	1.025	0.839	0.559	0.403	0.340	0.385	0.166	0.086	0.044
20cm		AVERAGE = 20.03	13.21	Averaged reading/V	0.048	0.090	0.223	0.789	1.012	1.035	1.223	1.176	1.326	1.629	1.639	1.237	1.025	0.840	0.559	0.403	0.340	0.385	0.166	0.087	0.046
	Attenuator	50.00		Horizontal offset W/mm	-1.00	-0.90	-0.80	-0.70	-0.60	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
	80º	50.10		Reading 1/V	0.004	0.012	0.018	0.016	0.084	0.284	0.640	1.112	1.217	2.406	2.563	2.564	2.054	1.319	0.959	0.312	0.099	0.027	0.008	0.008	0.003
		50.05		Reading 2/V	0.003	0.010	0.016	0.016	0.085	0.285	0.642	1.113	1.213	2.395	2.562	2.565	2.053	1.317	0.960	0.312	0.099	0.029	0.009	0.008	0.003
		50.05		Reading 3/V	0.003	0.010	0.015	0.016	0.084	0.284	0.638	1.111	1.218	2.403	2.564	2.564	2.052	1.316	0.959	0.314	0.099	0.029	0.009	0.008	0.002
50cm		AVERAGE = 50.05	43.23	Averaged reading/V	0.003	0.011	0.016	0.016	0.084	0.284	0.640	1.112	1.216	2.401	2.563	2.564	2.053	1.317	0.959	0.313	0.099	0.028	0.009	0.008	0.003
	Attenuator	100.00		Horizontal offset W/mm	-1.00	-0.90	-0.80	-0.70	-0.60	-0.50	-0.40	-0.30	-0.20	-0.10	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
	75º	100.00		Reading 1/V	0.042	0.122	0.235	0.260	0.656	1.058	1.527	1.625	2.030	2.451	2.570	2.216	1.589	1.088	1.087	0.385	0.198	0.079	0.038	0.020	0.006
		100.10		Reading 2/V	0.043	0.123	0.236	0.258	0.653	1.057	1.532	1.630	2.031	2.444	2.569	2.225	1.593	1.089	1.080	0.384	0.199	0.079	0.039	0.019	0.007
		99.95		Reading 3/V	0.042	0.122	0.234	0.259	0.657	1.057	1.533	1.627	2.033	2.449	2.570	2.219	1.593	1.090	1.082	0.385	0.198	0.078	0.038	0.019	0.005
1m	.	AVERAGE = 100.01	93.20	Averaged reading/V	0.042	0.122	0.235	0.259	0.655	1.057	1.531	1.627	2.031	2.448	2.570	2.220	1.592	1.089	1.083	0.385	0.198	0.079	0.038	0.019	0.006
	Attenuator	199.95		Horizontal offset W/mm	-2.00	-1.80	-1.60	-1.40	-1.20	-1.00	-0.80	-0.60	-0.40	-0.20	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40	1.60	1.80	2.00
	65º	199.95		Reading 1/V	0.019	0.038	0.076	0.158	0.306	0.540	0.935	1.358	1.814	2.217	2.277	2.030	1.552	1.063	0.703	0.278	0.111	0.060	0.020	0.012	0.002
		199.90		Reading 2/V	0.018	0.039	0.078	0.160	0.308	0.534	0.938	1.359	1.819	2.203	2.263	2.019	1.551	1.061	0.700	0.283	0.110	0.061	0.021	0.012	0.002
		199.95		Reading 3/V	0.019	0.039	0.079	0.160	0.312	0.540	0.933	1.360	1.824	2.219	2.270	2.032	1.553	1.062	0.704	0.279	0.111	0.060	0.021	0.011	0.002
2m		AVERAGE = 199.94	193.12	Averaged reading/V	0.019	0.039	0.078	0.159	0.309	0.538	0.935	1.359	1.819	2.213	2.270	2.027	1.552	1.062	0.702	0.280	0.111	0.060	0.021	0.012	0.002
	Attenuator	500.05		Horizontal offset W/mm	-4.00	-3.60	-3.20	-2.80	-2.40	-2.00	-1.60	-1.20	-0.80	-0.40	0.00	0.40	0.80	1.20	1.60	2.00	2.40	2.80	3.20	3.60	4.00
	45º	500.00		Reading 1/V	0.021	0.048	0.092	0.232	0.473	0.809	1.266	1.754	2.177	2.421	2.522	2.445	2.091	1.690	1.111	0.739	0.408	0.166	0.073	0.035	0.015
		500.00		Reading 2/V	0.021	0.049	0.093	0.237	0.478	0.796	1.268	1.755	2.180	2.434	2.514	2.446	2.090	1.689	1.106	0.735	0.404	0.165	0.071	0.033	0.016
		500.00		Reading 3/V	0.021	0.048	0.091	0.240	0.479	0.795	1.269	1.756	2.183	2.428	2.518	2.448	2.089	1.693	1.110	0.739	0.409	0.164	0.072	0.032	0.014
5m		AVERAGE = 500.01	493.20	Averaged reading/V	0.021	0.048	0.092	0.236	0.477	0.800	1.268	1.755	2.180	2.428	2.518	2.446	2.090	1.691	1.109	0.738	0.407	0.165	0.072	0.033	0.015
	Attenuator	1000.10		Horizontal offset W/mm	-5.00	-4.50	-4.00	-3.50	-3.00	-2.50	-2.00	-1.50	-1.00	-0.50	0.00	0.50	1.00	1.50	2.00	2.50	3.00	3.50	4.00	4.50	5.00
	25º	1000.15		Reading 1/V	0.301	0.476	0.692	0.981	1.300	1.611	1.931	2.222	2.498	2.660	2.755	2.727	2.626	2.394	2.132	1.804	1.495	1.184	0.905	0.658	0.466
		999.90		Reading 2/V	0.305	0.469	0.697	0.985	1.301	1.612	1.929	2.223	2.494	2.657	2.751	2.733	2.621	2.393	2.128	1.803	1.497	1.184	0.903	0.657	0.468
		1000.10		Reading 3/V	0.305	0.472	0.696	0.976	1.295	1.615	1.932	2.222	2.498	2.662	2.749	2.725	2.627	2.401	2.125	1.801	1.499	1.180	0.907	0.657	0.468
10m		AVERAGE = 1000.06	993.25	Averaged reading/V	0.304	0.472	0.695	0.981	1.299	1.613	1.931	2.222	2.497	2.660	2.752	2.728	2.625	2.396	2.128	1.803	1.497	1.183	0.905	0.657	0.467
	Attenuator	1499.90		Horizontal offset W/mm	-6.00	-5.40	-4.80	-4.20	-3.60	-3.00	-2.40	-1.80	-1.20	-0.60	0.00	0.60	1.20	1.80	2.40	3.00	3.60	4.20	4.80	5.40	6.00
	0º	1499.95		Reading 1/V	0.334	0.433	0.515	0.626	0.743	0.833	0.929	1.020	1.080	1.107	1.118	1.108	1.070	1.003	0.918	0.820	0.707	0.604	0.497	0.404	0.312
		1499.95		Reading 2/V	0.332	0.434	0.516	0.625	0.742	0.833	0.928	1.018	1.084	1.108	1.120	1.107	1.067	1.003	0.922	0.819	0.703	0.605	0.497	0.405	0.313
		1500.00		Reading 3/V	0.333	0.437	0.522	0.628	0.743	0.831	0.928	1.022	1.085	1.109	1.170	1.108	1.070	0.999	0.920	0.822	0.700	0.608	0.503	0.405	0.314
15m		AVERAGE = 1499.95	1493.13	Averaged reading/V	0.333	0.435	0.518	0.626	0.743	0.832	0.928	1.020	1.083	1.108	1.136	1.108	1.069	1.002	0.920	0.820	0.703	0.606	0.499	0.405	0.313

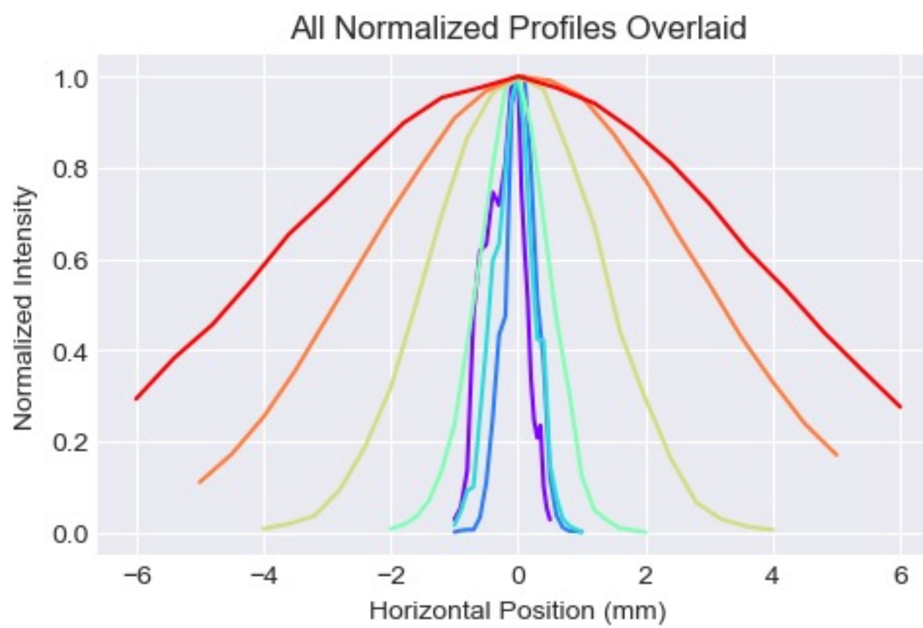
All Gaussian Intensity Profiles with Attenuator Correction Applied



Correction Method:

$$I_{actual} = \frac{I_{measured}}{T}, \text{ where } T = \cos\vartheta_{attenuator}$$

Distance (Attenuator, Transmission)
20 cm (θ= 85°, T=0.008)
50 cm (θ= 80°, T=0.030)
100 cm (θ= 75°, T=0.067)
200 cm (θ= 65°, T=0.179)
500 cm (θ= 45°, T=0.500)
1000 cm (θ= 25°, T=0.821)
1500 cm (θ= 0°, T=1.000)



Rayleigh Range Z_R

$$\frac{W^2}{W_0^2} = 1 + \frac{Z^2}{Z_R^2}$$

The spread of the spot size W(z) is hyperbolic.

$$W^2 = \left((0.257 \pm 0.008) \times 10^{-6}\right) Z^2 + 0.176 \times 10^{-6} \text{ mm}^2$$

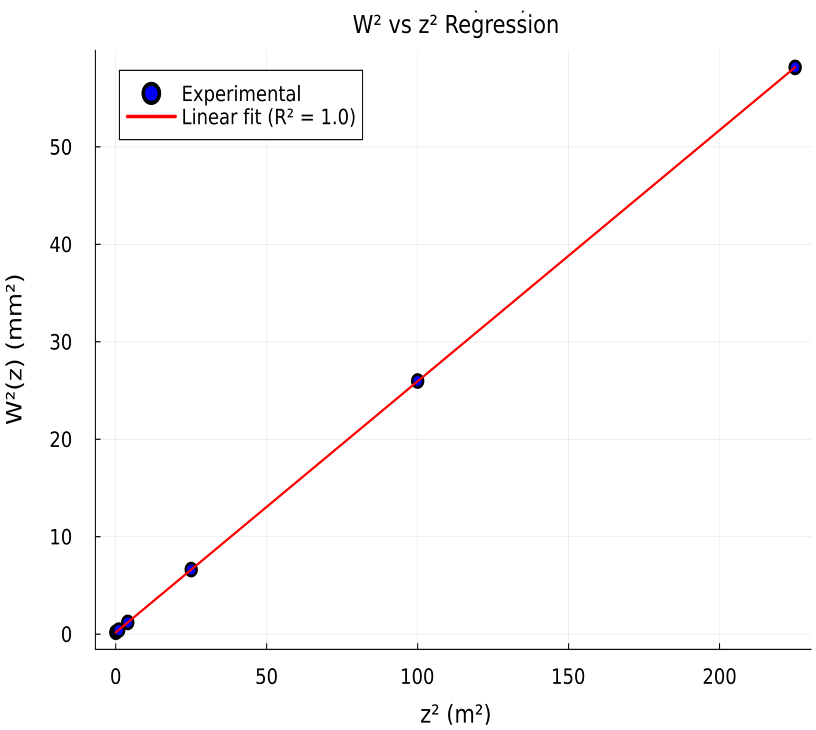
Now,

$$W_0 = \sqrt{c} = \sqrt{0.176 \times 10^{-6} \text{ mm}^2} \implies W_0 = 0.42 \pm 0.011 \text{ mm}$$

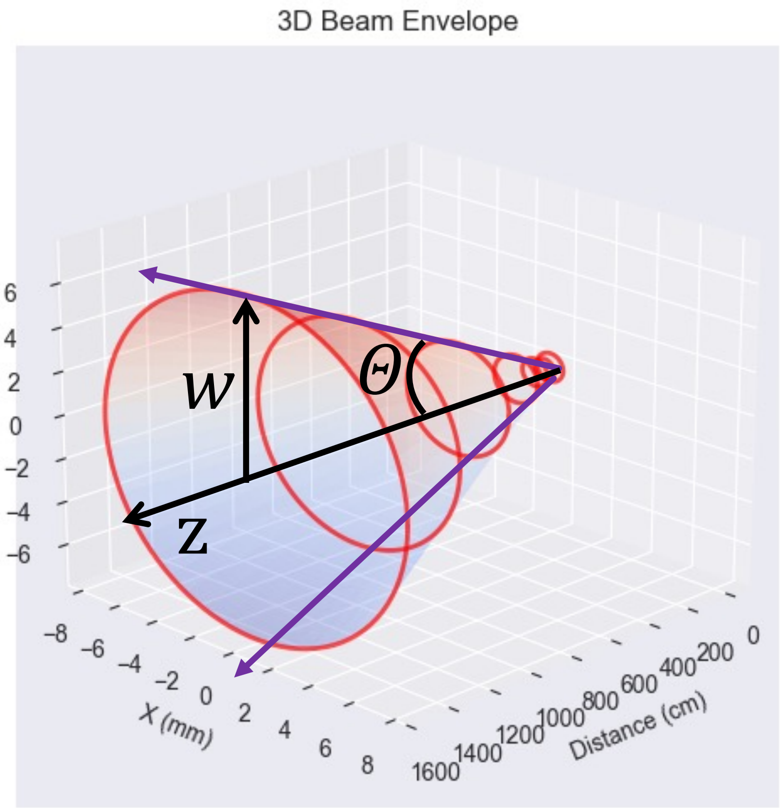
$$Z_R = \sqrt{\frac{c}{m}} = \sqrt{\frac{0.176 \times 10^{-6} \text{ m}^2}{0.257 \times 10^{-6}}} \implies Z_R = 0.827 \pm 0.038 \text{ mm}$$

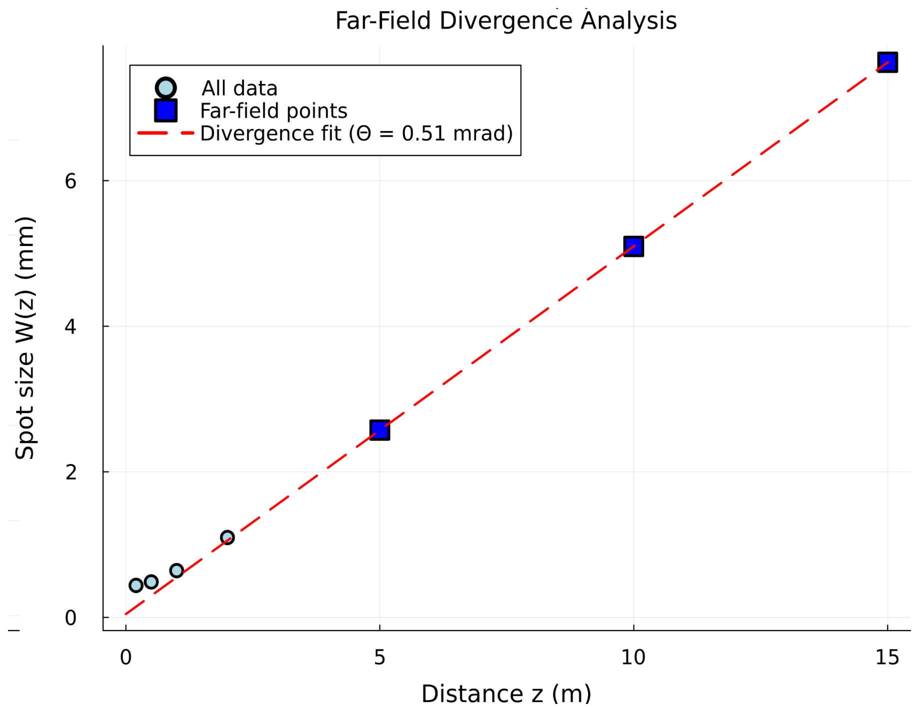
Consequently,

$$\theta = \sqrt{\frac{\lambda}{\pi Z_R}} = \sqrt{\frac{632.8 \times 10^{-6}}{3.1416 \times 0.827}} \implies \boxed{\theta = 0.458 \pm 0.055 \text{ mrad}}$$



Determination of $\boldsymbol{\Theta}$ (Far Field)



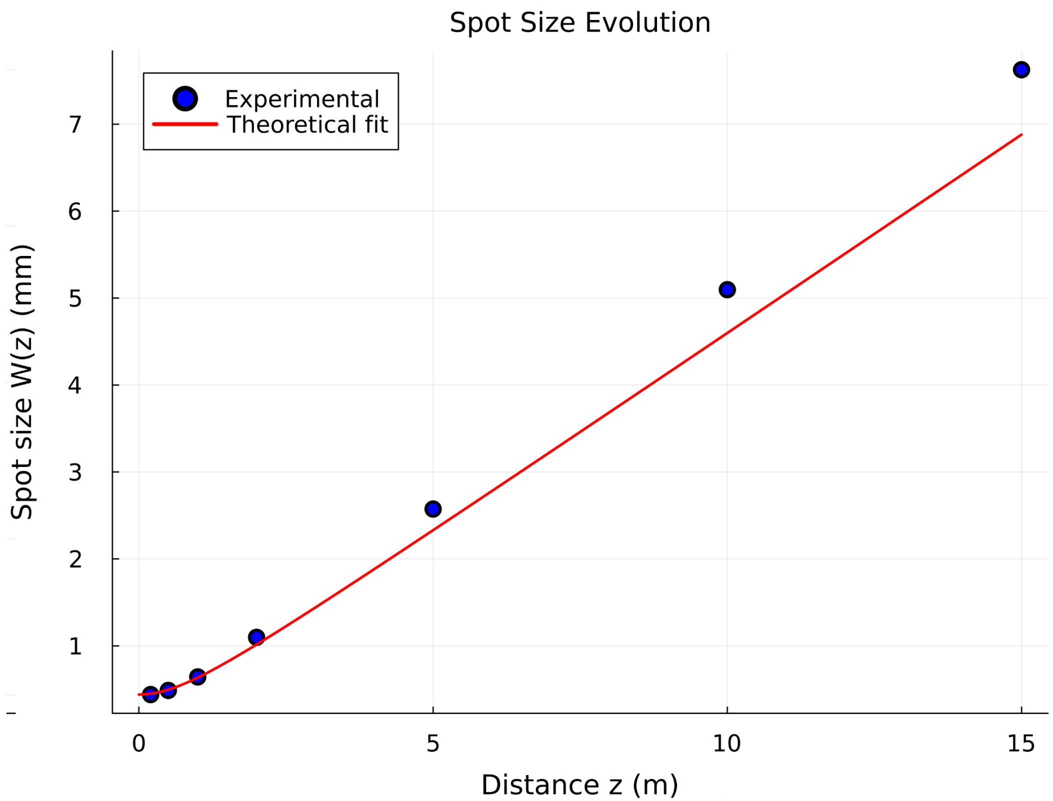


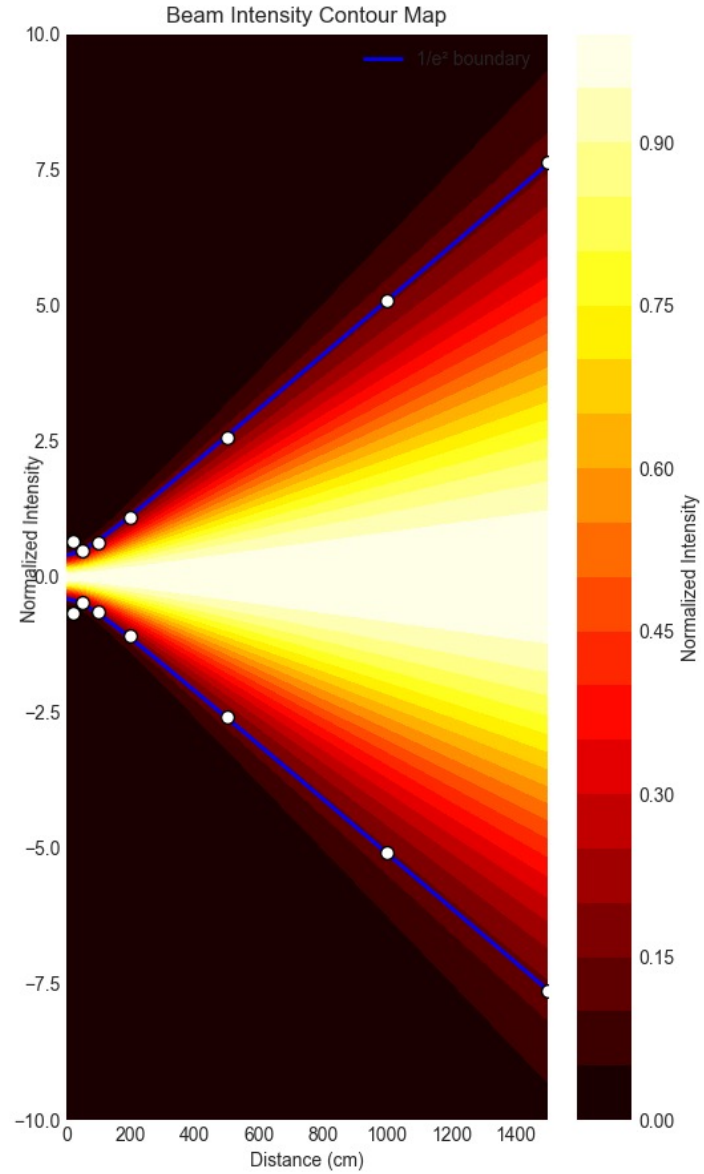
Using distances: [5.0, 10.0, 15.0] m

Slope ($\Theta_{\text{experimental}}$): $\boxed{0.505 \pm 0.016 \text{ mrad}}$

Intercept: 0.046 mm

M^2 parameter of laser





$$M^2 = \frac{\Theta}{\theta} = \frac{0.505}{0.458} \implies \boxed{M^2 = 1.104 \pm 0.068}$$

Error Analysis

In all the calculation in this experiment involve either measuring the position-distance or the voltage of the photon detector.

- For the micrometer stage mounted detector the error $\varepsilon = \frac{0.01}{1} \approx 1\%$
- For the measurement of the distance Z:
  - for the close range $\varepsilon = \frac{1}{50} \approx 2\%$
  - for far range $\varepsilon = \frac{0.10}{10} \approx 1\%$
- The error in voltage of the photon detector
  - for the close range $\varepsilon = \frac{0.05}{5} \approx 1\%$
  - for far range $\varepsilon = \frac{0.05}{1.5} \approx 3\%$

Error Analysis

Consequently:

- For the determination of the Z_R the error from the line fit of 4.2% is the biggest error.
- For W_0 error from the line fit 1.8% so the ε_V is the biggest error
- For the determination of θ error from the line fit is 1.4% so ε_V is the biggest error

Finally from the law of error propagation $\varepsilon_M = \sqrt{\varepsilon_\theta^2 + \varepsilon_{Z_R}^2 + \varepsilon_{W_0}^2} = 5.97\%$

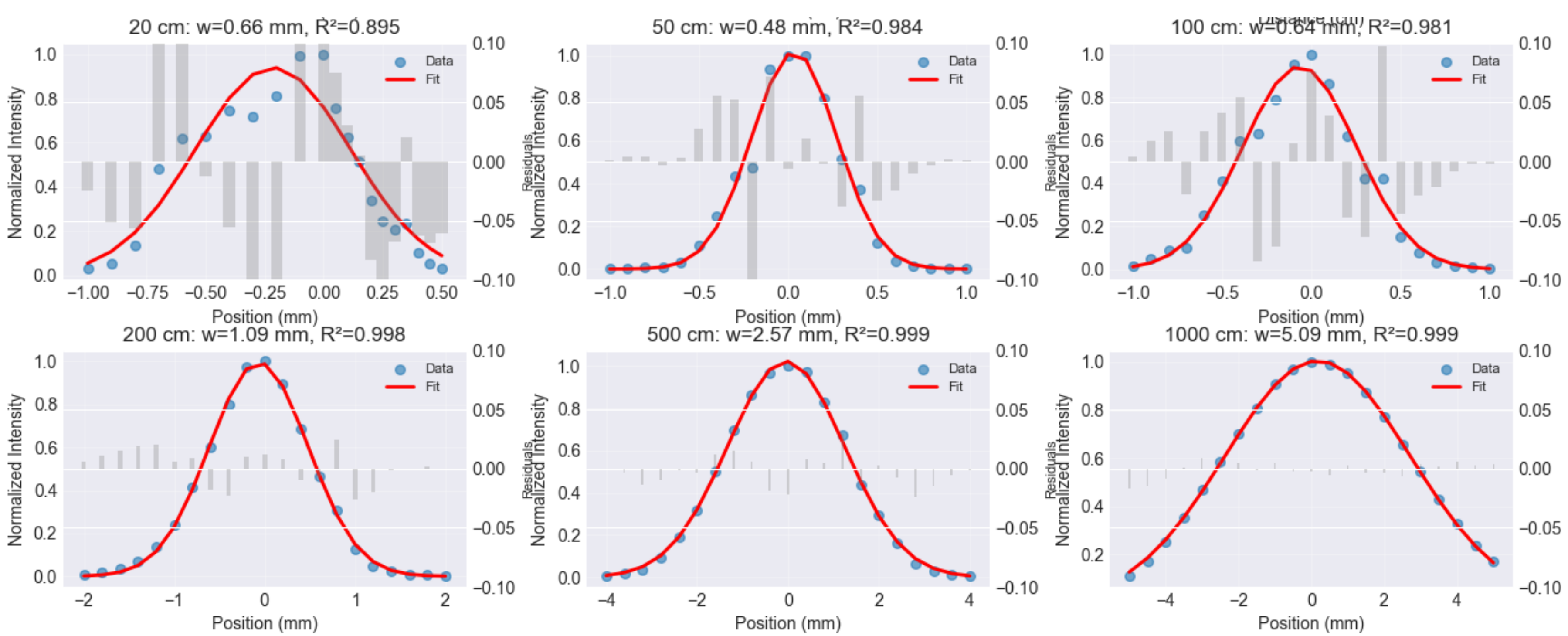
Conclusion

- The measured intensity profile confirmed that the laser used in the experiment follows the laws of a Gaussian intensity distribution.
- The M^2 factor of the laser was measured as $M^2 = 1.104 \pm 0.068$, which is only a 10% deviation from the ideal value of $M^2 = 1$. This indicates the laser has good beam quality with minimal deviation from an ideal Gaussian beam.
- The found value of M^2 deviates by 9.9% from the manufacturer's claimed value of less 1.05.

Sources of Error

Detector is manually placed and alignment is ensured by only checking the reflection of beam with our eyes.

The voltmeter saturated very quickly and that led us to take many sets of wrong readings



Thank You!

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Open for Questions